Teaching Elementary Mathematics— Urmila Devi Dasi

Ideally, young children should be exposed to mathematical concepts from the time they can manipulate objects. Ideas such as bigger and smaller, and more and less, form during that time. As a child grows, parents should speak to him or her about the mathematics of everyday life—how many plates should be put on the table, how many cups of flour go in the cake, how many inches wide the bookcase needs to be, how many miles have been covered in the trip, and how many gallons of gas the car uses each mile.

Children who are frequently exposed to such conversation, related to what they are experiencing at the moment, develop an internal “sense” of math. They know what five “means” and have a huge advantage as they enter school. For such children, their first mathematics instruction can be with simple manipulatives, discussion, pictures, and worksheets. The hardest thing for them to grasp is usually place value, as it is rare that a child has any but the vaguest idea of such a concept before entering kindergarten. While base ten blocks are one of the best materials for teaching place value, in order to learn with them, children usually have to again and again get out the correct blocks in response to a number, and give the correct number when seeing blocks. It can take months to understand that a 2 in one place means quite a different thing from a 2 in another.

Children without much or any prior mathematics base can take anywhere from one to four years to master the basic sense of numbers. So, when moving ahead with teaching “carrying” in addition or “borrowing” in subtraction, the teacher must always be alert to any indication that the child is acting from rote memorization rather than understanding. Such children need a tremendous amount of exposure to manipulatives, and should only work problems exclusively with paper and pencil when they consistently display a through understanding.

While the published texts that are best for teaching math in the primary grades, such as Miquon, use extensive derivation rather than calculation and algorithms, it is often the children who would benefit most from this approach—those with little prior knowledge—who find it challenging and frustrating. We wish all children could grasp that adding nine means one less than adding ten, but find that even repeated exercises with concrete objects and seeing a pattern is insufficient for some children, who may make it more difficult for themselves by insisting on learning through brute rote memorizing.

Children who’ve been exposed to math in their early years, or who are able to gain the same understanding in the first months or years of school, find great excitement in the discovery of the reliable and logical patterns that allow math to be an exact science.

It’s fairly obvious which children are learning the concepts and which are rote memorizing. Understanding is demonstrated when a child can explain or show the how and why of an answer. It is also shown when a child knows when to apply a learned formula. Children who have used brute memory to “learn” math will often or consistently apply a process wrongly or become confused when a problem has a slightly different wording or form. Sometimes a child's difficulty with a simple procedure will be apparent when he or she cannot understand a more advanced process. For example, a child who cannot grasp how to add fractions with unlike denominators may betray his or her lack of understanding of the meaning of the more foundational process of finding equivalent fractions.

I attempt to teach, therefore, by having understanding precede practice and memorization. Some math experts maintain that through repeated practice without understanding, understanding will come. I do remember learning place value in such a way when I was about six years old and have experience of this method working for older students. Yet, I have also seen many older students struggle or even fail because the instability of years of memorizing without understanding caused their problem-solving knowledge to tumble.

Finally, it is necessary for the child to form a mental "bridge" between concrete and abstract understanding. Merely using manipulatives doesn't guarantee that this bridge exists. Generally, the first step is to have the children use blocks, etc. and then blocks while the teacher writes. Next blocks with the student writing, then writing only. It is best if this procedure can then be repeated with another type of object.

With the principle that understanding should precede or at least accompany memorization, we start the beginning student with counting. The child needs to know the relationship between the verbal numeral, written numeral, and number of objects. We therefore have him or her relate tangible objects with numerals, either that are pre-written or that the child writes. There should be much practice with seeing a two or three-digit numeral, saying it, writing it, and getting out the corresponding blocks.

While this process is being mastered, the child engages in simple addition. Sometimes this is done by comparing different groups of blocks, and sometimes by using a scale. In a similar way, the children gradually learn measurement, fractions, time, estimation, and so on. Hopefully, it's not necessary to list the standards or general scope and sequence expected for various grade levels, as such information is easily available from the National Council of Teachers of Mathematics.

I continue to have regular review of all mathematical principles and operations as we are learning new skills. I follow this policy of regular, ongoing review right through high school. It is essential that children work the same types of problems many times long after they've understood the concept. The reason for this is to develop an "automatic reaction"—the child has done so many computations of distance, for example, that he or she could practically answer such questions without thinking much about it. The child is so competent that he or she no longer thinks about, or perhaps even consciously remembers, the full why and process. Such "automaticity" can be compared to the way we drive a car once we've been driving for many years. We're hardly consciously aware of our driving processes.

Once a child has grasped the essential concepts—usually by the end of third or fourth grade—we use manipulatives only when visual representation, verbal explanations, and practice still leave a child confused. For example, I recently helped a sixth grade boy who said he couldn't "get" multiplying fractions. Multiplying fractions is one of the simplest operations—one multiplies the numerators and multiplies the denominators. Doing so usually makes more sense to a child than does adding fractions. Children always want to add the denominators and usually need much time and thought before they understand that ½ and ¼, for example, are not the same types of "thing."

In this case, I demonstrated the reason of the process by using the principle that multiplication can imply area. In other words, if one is multiplying 3 times 4, one can build a rectangle that is 3 in width and 4 in length. In the same way, using some ingeniously developed fraction squares, we were able to see and feel that 2/6 times 1/3 was, indeed, an area that was 2/6 length and 1/3 width. We were also able to have tangible evidence that the 1/3 "cut" the 2/6 pieces each into three. It was interesting to the student to grasp that multiplying with fractions gave him a smaller number, as he had assimilated the idea that multiplying means enlarging. We discussed, drawing diagrams and using the manipulatives, how multiplying by 1/3 really means dividing by three. Both he and I supplied several real life examples of multiplying with both whole numbers and fractions.

Perhaps the most difficult and rewarding aspect of teaching mathematics to elementary students is helping them to develop systems of derivation rather than calculation. I **always** teach the mental process of derivation—sometimes in several ways—before teaching a calculation formula. Of course, to derive answers successfully, children have to not only have a sense of numbers and their relationships, but be willing to risk thinking in a way outside of the textbook or other than what their parents model when they do their homework.

Mathematics is exciting—the logic and interrelationship of systems, the thrill of understanding how objects fits together, and knowledge that allows us to facilitate our life's goals. When children grasp these elements and apply them in their other studies and their lives outside of schoolwork, we feel truly successful.